Hierarchy of critical behaviours in the interacting three colouring model on the honeycomb lattice

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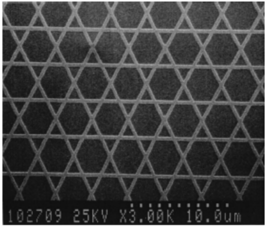


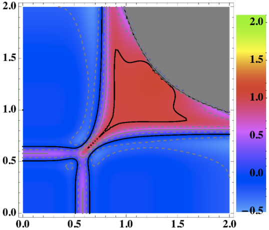
Pierre Pujol

LPT IRSAMC

Université Paul Sabatier

Jacob Simmons

JFI, University of Chicago



Open Statistical Physics, The Open University, Milton Keynes UK, March 2, 2011

Outline

- (semi) historical introduction to the three-colouring model
- the role of interactions: a “continuously varying” central charge in a unitary system?
- further insight introducing colour-dependent coupling constants
- conclusions and open questions

The model

Baxter 1970

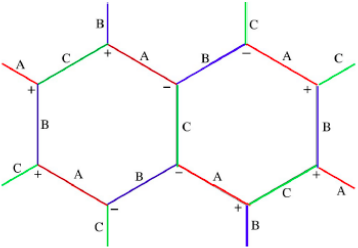
JOURNAL OF MATHEMATICAL PHYSICS VOLUME 11, NUMBER 3 MARCH 1970

Colorings of a Hexagonal Lattice

R. J. BAXTER
Mathematics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 13 May 1969)

The number of ways W^L of coloring the bonds of a hexagonal lattice of L sites (L large) with three colors so that no adjacent bonds are colored alike is calculated exactly, giving $W = 1.20872\cdots$. This is equivalent to counting the number of 4-colorings of the faces of the lattice and can also be regarded as a multiple-dimer problem. If one introduces activities corresponding to certain vertex configurations, then the system is found to have an infinite-order phase transition between two ordered states.

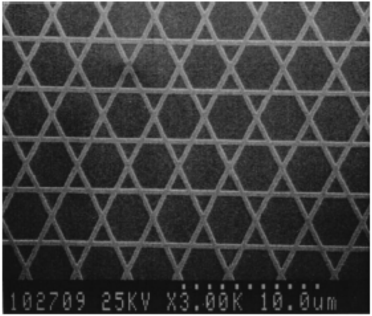


- hard constraint: no two bonds with equal colour can meet at any vertex
- extensive degeneracy
- power-law correlations

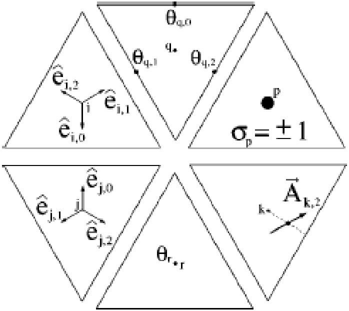
$SU(3)_{k=1}$ WZNW CFT N. Read

Experimental realisations

Higgins et al. 2000, Castelnovo et al. 2004



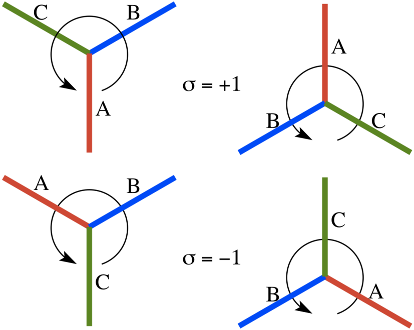
Higgins *et al.* 2000



Castelnovo *et al.* 2004

The role of interactions

Park + Huse 2001, Castelnovo et al. 2004



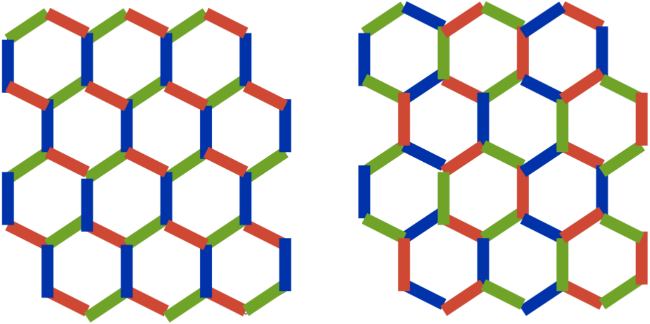
$$\sigma_i = \begin{cases} +1 & \text{even counter-clockwise permutation} \\ -1 & \text{odd counter-clockwise permutation} \end{cases}$$

$$E = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

- glassy dynamics (freezing into polycrystalline metastable states) [Phys. Rev. B **69**, 104529 (2004)]
- unusual critical behaviour

The role of interactions

Park + Huse 2001, Castelnovo et al. 2004

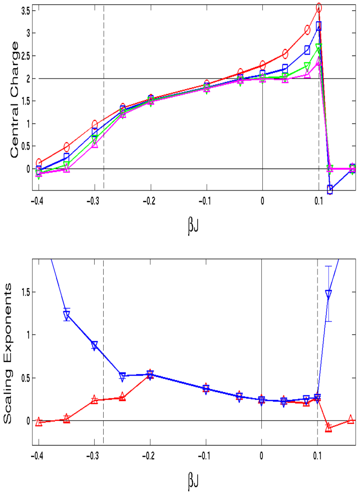


$J \to +\infty$ $\qquad$ $J \to -\infty$

- $J > 0 \longrightarrow$ favours parallel bonds having the same colour
- $J < 0 \longrightarrow$ favours alternating sequences of two colours around hexagonal plaquettes

Behaviour of the central charge

Castelnovo et al. 2006



- $J = 0 \longrightarrow SU(3)_{k=1}$ Wess-Zumino-Novikov-Witten conformal field theory (central charge $c = 2$)
- $J > 0 \longrightarrow$ line of critical points ending in a first order transition (approx. constant c and scaling exponents)
- $J < 0 \longrightarrow$ *continuously varying central charge?*

Colour-dependent coupling constants

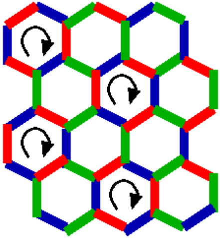
CC+P.Pujol+J.Simmons

Kolmogorov ‘c’ theorem (for a unitary CFT) *forbids* a continuously varying central charge!

new insight from colour-dependent couplings (A=red, B=blue, C=green):

$$E = -\sum_{\langle ij \rangle} J_{\alpha_{ij}} \sigma_i \sigma_j \qquad \alpha_{ij} = A, B, C$$

- $J_A = J_B = -\infty \longrightarrow$ Ising model on the triangular lattice with nn interaction J_C
- $J_A = -\infty,\ J_B = J_C \equiv J \longrightarrow$ dimer model on the honeycomb lattice with loop tension J Jacobsen+Alet 2009



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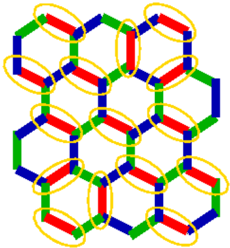

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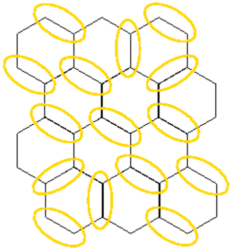
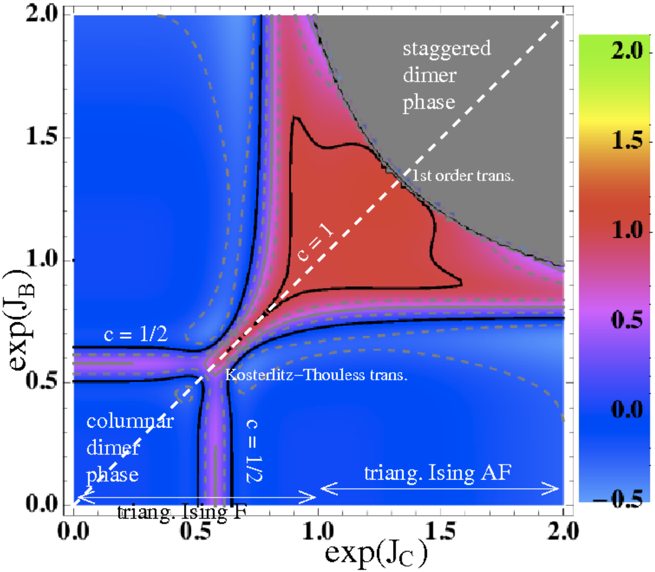


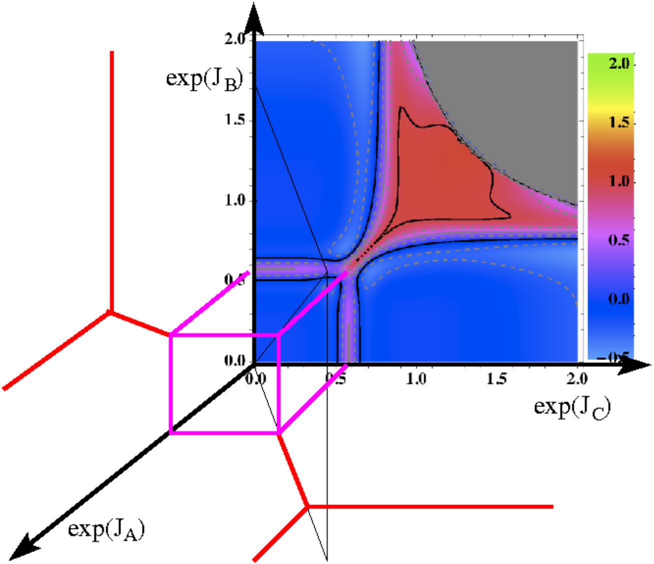
The $J_A = -\infty$ plane

CC+P.Pujol+J.Simmons



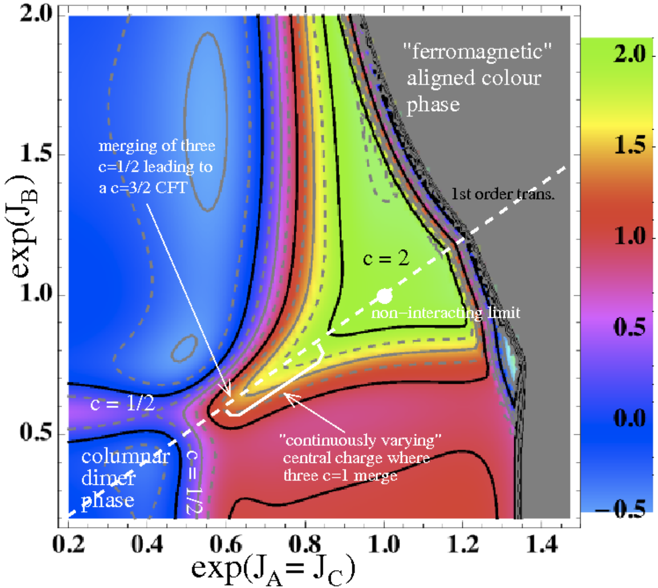
The full phase diagram

CC+P.Pujol+J.Simmons



The $J_A = J_C$ plane

CC+P.Pujol+J.Simmons



Conclusions and open questions

- competition between short-range interaction and hard-constraints $\Rightarrow$ wide range of critical behaviours

- new insight on how to interpret the different critical behaviours comes from colour-dependent couplings:
  - two $c = 1/2$ (free majoranas) merge into a $c = 1$ (free boson)
  - three $c = 1/2$ (free majoranas) merge into a $c = 3/2$ *as if they were independent* (!) [supersymmetric point?]
  - two $c = 1$ (free bosons) merge into a $c = 2$ (free boson)
  - three $c = 1$ (free bosons) merge into $c \in (3/2, 2)$

EPSRC
Engineering and Physical Sciences Research Council

the paradox of the "continuously varying" central charge remains unsolved

SEPnet
South East Physics Network