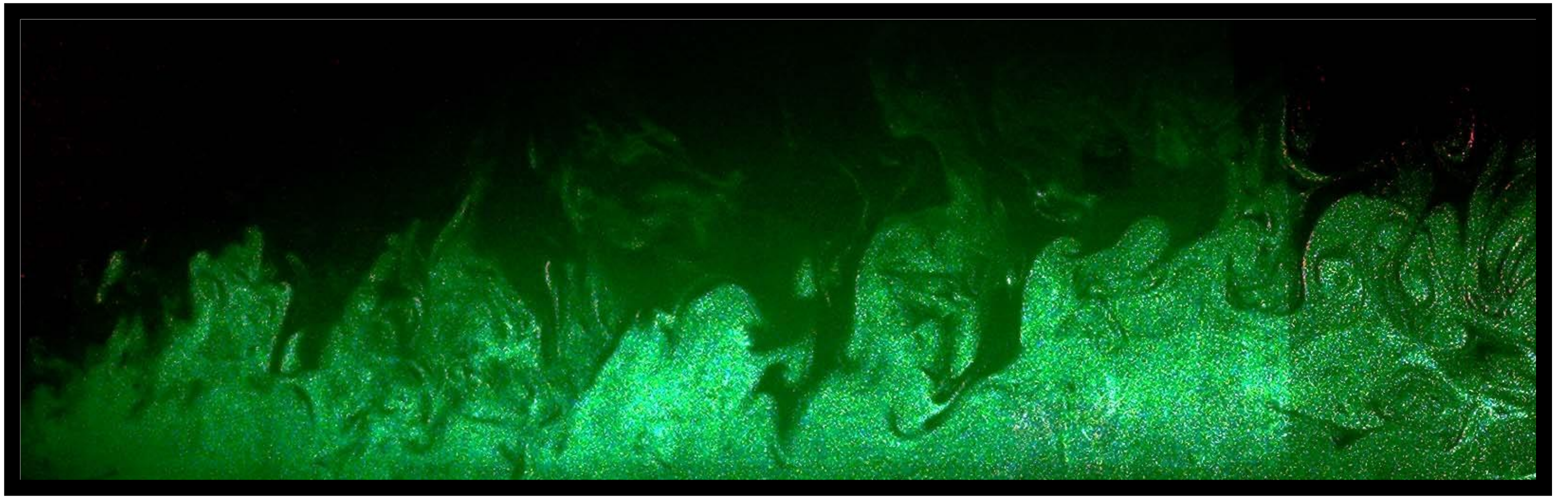
Non-local closure model for particle dispersion tensors in a turbulent boundary layer



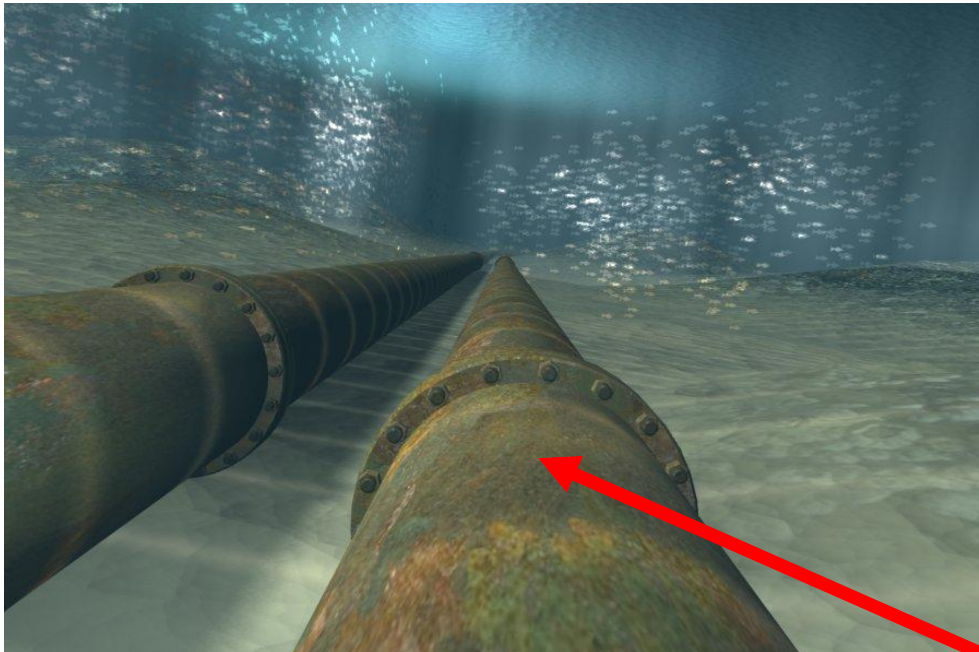
Andy Bragg

Newcastle University

Overview of talk

- Motivation for research
- Brief introduction to pdf kinetic equation and its dispersion tensors
- Why a closure model is needed for the dispersion tensors
- New closure model: ***non-local, accounts for effects of turbulence inhomogeneity, drag, added mass, gravity and particle-wall collisions*** on the dispersion tensors
- Conclusions & Future work

Motivation for Research



Sand Particles – Pipe erosion

Wax Particles – Deposit on walls of pipelines which restricts flow

Hydrate Particles – Can form plugs which block pipelines

PDF Kinetic Equation

$$\frac{\partial}{\partial t}p = -\mathrm{v}_i\frac{\partial}{\partial \mathrm{x}_i}p - \frac{\partial}{\partial \mathrm{v}_i}\left[(\mathrm{F}_i + \kappa_i)p\right] + \frac{\partial}{\partial \mathrm{v}_j}\left[\frac{\partial}{\partial \mathrm{x}_i}\left[\lambda_{ij}p\right] + \frac{\partial}{\partial \mathrm{v}_i}\left[\mu_{ij}p\right]\right]$$

Particle Dispersion Tensors

- By integrating the pdf equation over velocity space the particle continuum equations are obtained: their solutions give the mean particle concentration, mean particle velocity, mean particle Reynolds stresses…

Particle Dispersion Tensors

- In the continuum equations we have the velocity averaged dispersion tensors:

$$\overline{\lambda}_{ij}(\mathbf{x},t) = \int_0^t \mathrm{G}_{im}(t;t')\left\langle R_{mj}(\boldsymbol{x}^{p\prime},t';\mathbf{x},t)\right\rangle_{\boldsymbol{x}^p(t)=\mathbf{x}} \; dt'$$

$$\overline{\mu}_{ij}(\mathbf{x},t) = \int_0^t \dot{\mathrm{G}}_{im}(t;t')\left\langle R_{mj}(\boldsymbol{x}^{p\prime},t';\mathbf{x},t)\right\rangle_{\boldsymbol{x}^p(t)=\mathbf{x}} \; dt'$$

$$\overline{\kappa}_{i}(\mathbf{x},t) = \int_0^t \mathrm{G}_{nm}(t;t')\left\langle \frac{\partial}{\partial x_n} R_{mi}(\boldsymbol{x}^{p\prime},t';\mathbf{x},t)\right\rangle_{\boldsymbol{x}^p(t)=\mathbf{x}} \; dt'$$

$t' \leq t$

$\boldsymbol{R}$ - Eulerian 2-point, 2-time correlation tensor of turbulent force field

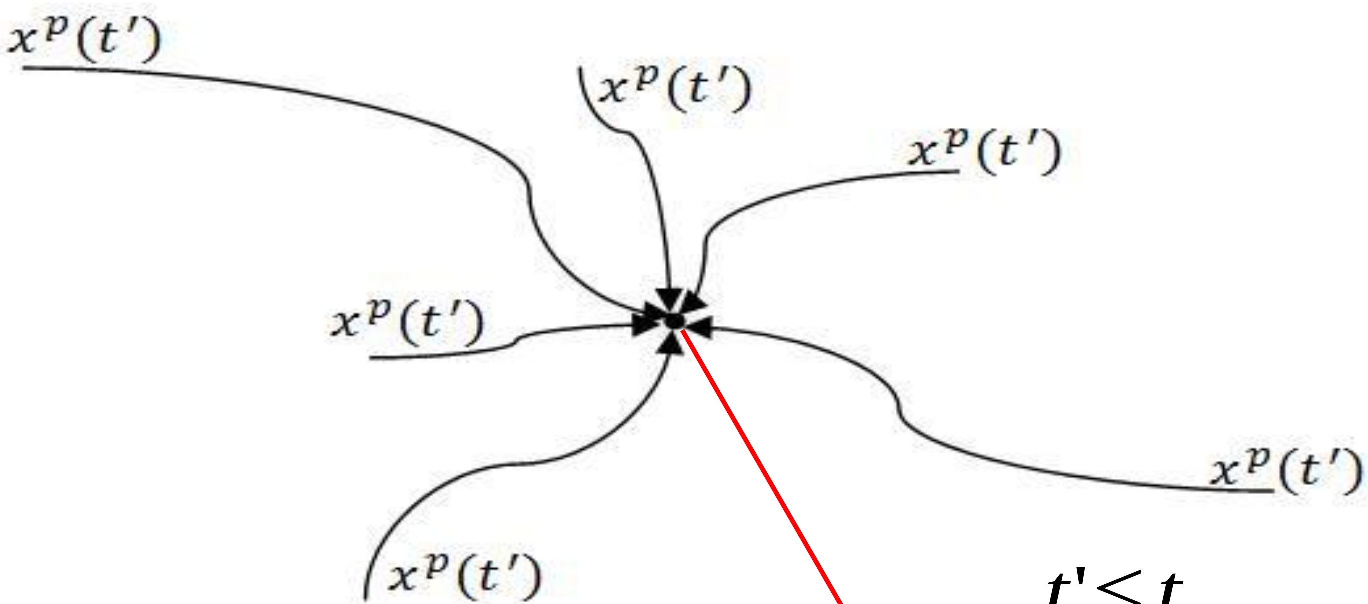
$\mathbf{x}$ - Position phase-space variable

$\boldsymbol{x}^p$ - Particle position $\quad$ $\boldsymbol{x}^{p\prime}$ - Particle position at time t'

Turbulence Correlation experienced by inertial particles

Unknown correlation tensor in particle dispersion tensors

$$\left\langle R_{mj}(\boldsymbol{x}^{p\prime}, t'; \mathbf{x}, t) \right\rangle_{\boldsymbol{x}^p(t)=\mathbf{x}}$$



$t' \leq t$

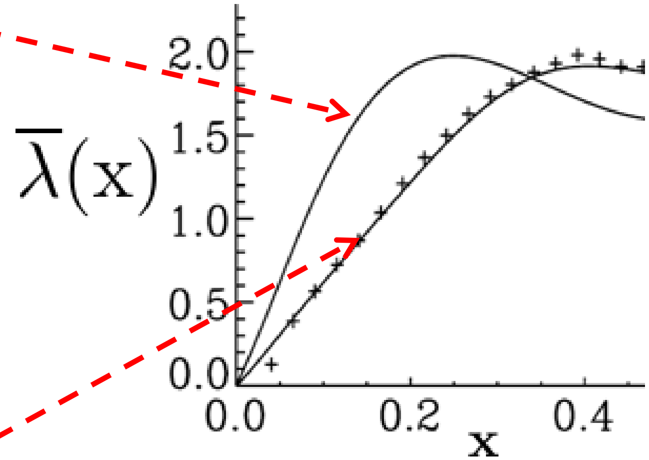
Particle trajectories converging to x at time t

Local Homogeneous Approximation (LHA)

- Usual approach to close the unknown correlation tensor is LHA:

$$\langle R_{ij}(\boldsymbol{x}^{p\prime}, t'; \mathbf{x}, t)\rangle_{\boldsymbol{x}^p(t)=\mathbf{x}} \approx R_{ij}(\mathbf{x}, t; \mathbf{x}, t)\exp\left(\frac{-|t-t'|}{\tau_L^p(\mathbf{x})}\right)$$

LHA



Particle Tracking data: the ‘real’ answer

Roar Skartlien, ‘Kinetic modelling of particles in stratified flow – Evaluation of dispersion tensors in inhomogeneous turbulence’ I.J.M.F (2007)

Why is LHA not sufficient to close $\left\langle R_{mj}(\boldsymbol{x}^{p\prime}, t'; \mathbf{x}, t) \right\rangle_{\boldsymbol{x}^p(t)=\mathbf{x}}$?

- Does not account for the fact that the particles are dispersing through a strongly inhomogeneous turbulent flow

- Does not account for the effect of particles moving into negatively correlated regions of the flow field

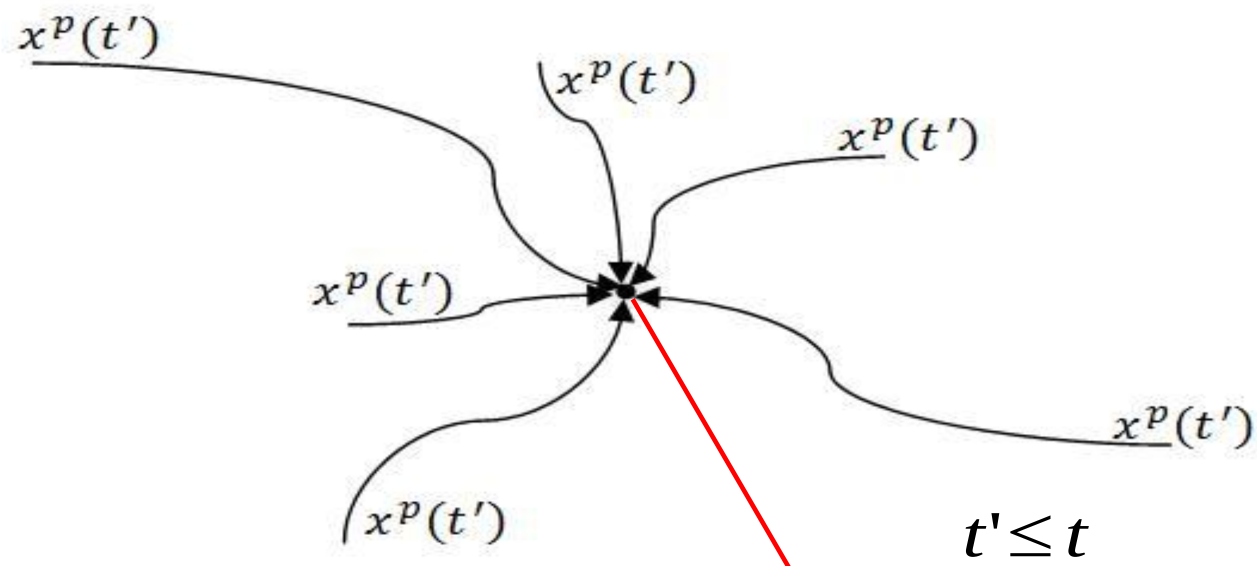
- Does not account for the effect of particle wall collisions on the correlations the particles see in the near wall region

New Non-local closure model

Single Phase Fluid data

$$\left\langle R_{ij}(\boldsymbol{x}^{p\prime}, t'; \mathbf{x}, t) \right\rangle_{\boldsymbol{x}^p(t)=\mathbf{x}} = \int_{\mathbf{x}'} R_{ij}(\mathbf{x}', t'; \mathbf{x}, t)\, \rho(\mathbf{x}', t' | \mathbf{x}, t)\, d\mathbf{x}'$$

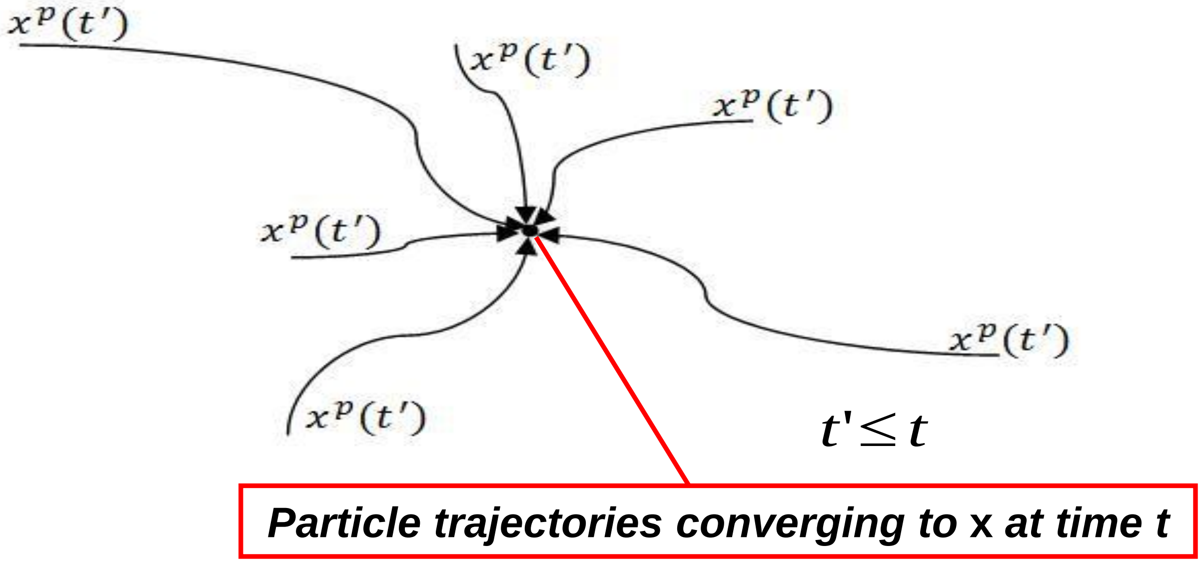
Distribution needs to be modelled



Particle trajectories converging to x at time t

Backwards in time dispersion

- The spatial distribution in the dispersion tensors is ‘backward in time’



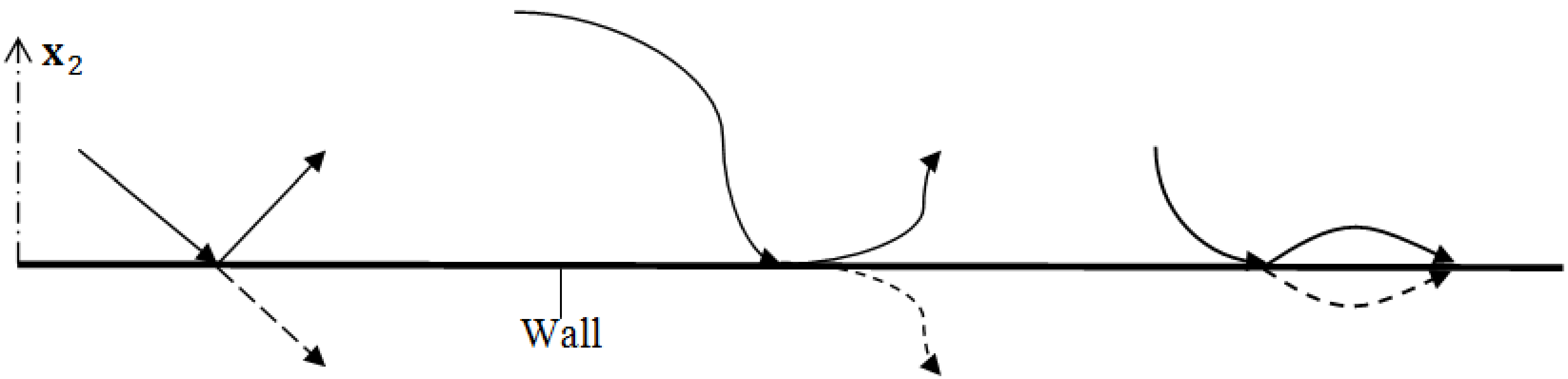
- Modelling the backward in time distribution is more difficult than modelling a forward in time distribution, but they can be related by Bayes’ theorem...

$$\rho(\mathrm{x}', t' | \mathrm{x}, t) = \rho(\mathrm{x}, t | \mathrm{x}', t') \frac{\rho(\mathrm{x}', t')}{\rho(\mathrm{x}, t)} \quad \underset{s = |t - t'|}{\overset{\text{Steady State}}{\Longrightarrow}} \quad \rho(\mathrm{x}', s | \mathrm{x}) = \rho(\mathrm{x}, s | \mathrm{x}') \frac{\rho(\mathrm{x}')}{\rho(\mathrm{x})}$$

Line of symmetry: to account for particle-wall collisions

- Only considering *elastic* particle-wall collisions

- The wall can be thought of as a line of symmetry for the particle dispersion process:



- Turbulence statistics are made symmetric about the wall

Constructing the moments of $\rho(\mathrm{x}, s|\mathrm{x}')$

$$\rho(\mathrm{x}, s|\mathrm{x}') = \left\langle \delta\left(\mathbf{x} - \boldsymbol{x}^p(s)\right) \middle| \delta\left(\mathbf{x}' - \boldsymbol{x}^p(0)\right) \right\rangle$$

$\langle x_i^p(s)\rangle$, $\langle x_i^p(s) x_j^p(s)\rangle$ *etc* can be obtained from

$$x_i^p(s) = G_{ij}(0) v_j^p(0) + \int_0^s G_{ij}(s - s_1) F_j\left(\boldsymbol{x}^p(s_1), s_1\right) ds_1$$

By constructing the moments in this way, a range of forces can be accounted for, such as drag, gravity, added mass etc, provided that the necessary statistics (mean, variance, skewness, autocorrelations etc) can be approximated

What type of distribution for $\rho(\mathrm{x}, s|\mathrm{x}')$?

- First Step – Gaussian distribution. But a Gaussian may not be sufficient…

- If Gaussian pdf proves insufficient then one of the 'skew family' will be used; i.e. Skew Normal Distribution, Skew t distribution etc

Conclusions and Future work

$$\overline{\lambda}_{ij}(\mathbf{x}, t) \qquad \overline{\mu}_{ij}(\mathbf{x}, t) \qquad \overline{\kappa}_i(\mathbf{x}, t)$$

Significance: we now have a ***non-local*** model for these dispersion tensors which takes into account the effects of ***turbulence inhomogeneity (and anisotropy), particle-wall collisions, drag, added mass and gravity***

A general closure model for this dispersion tensor has been derived. Previously, only approximations in the limit of very small particles were obtainable

- In the process of obtaining particle tracking data against which to test the new non-local closure models for the particle dispersion tensors.

Thank you for listening.

Questions?