Painlevé Transcendents and the Information Theory of MIMO Wireless Communication Systems

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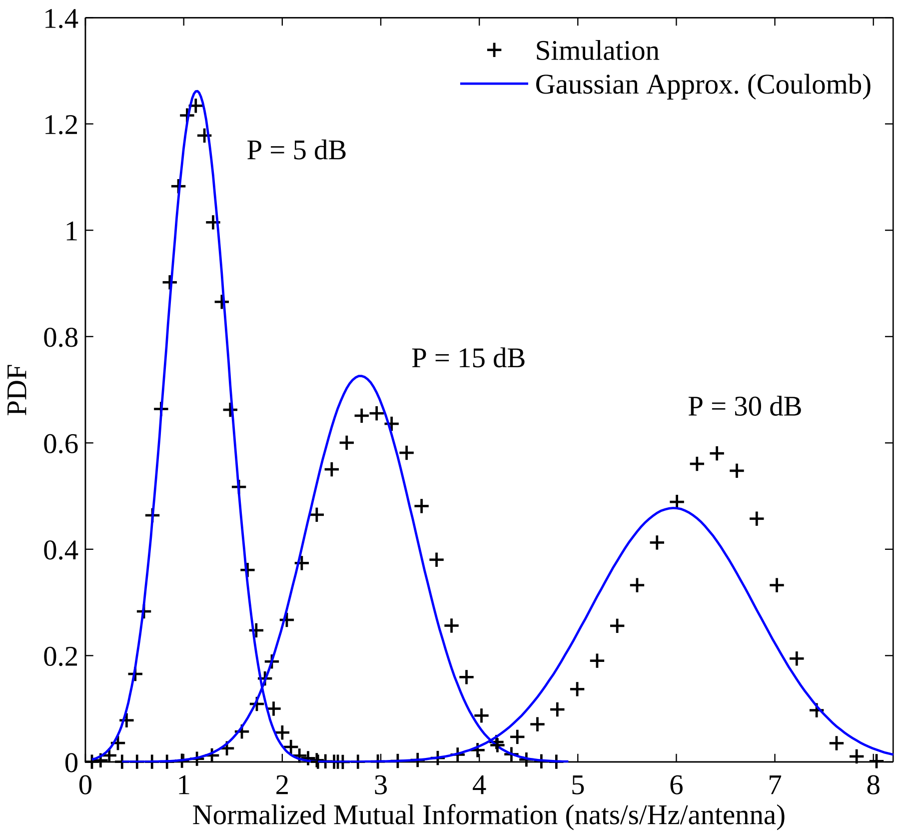
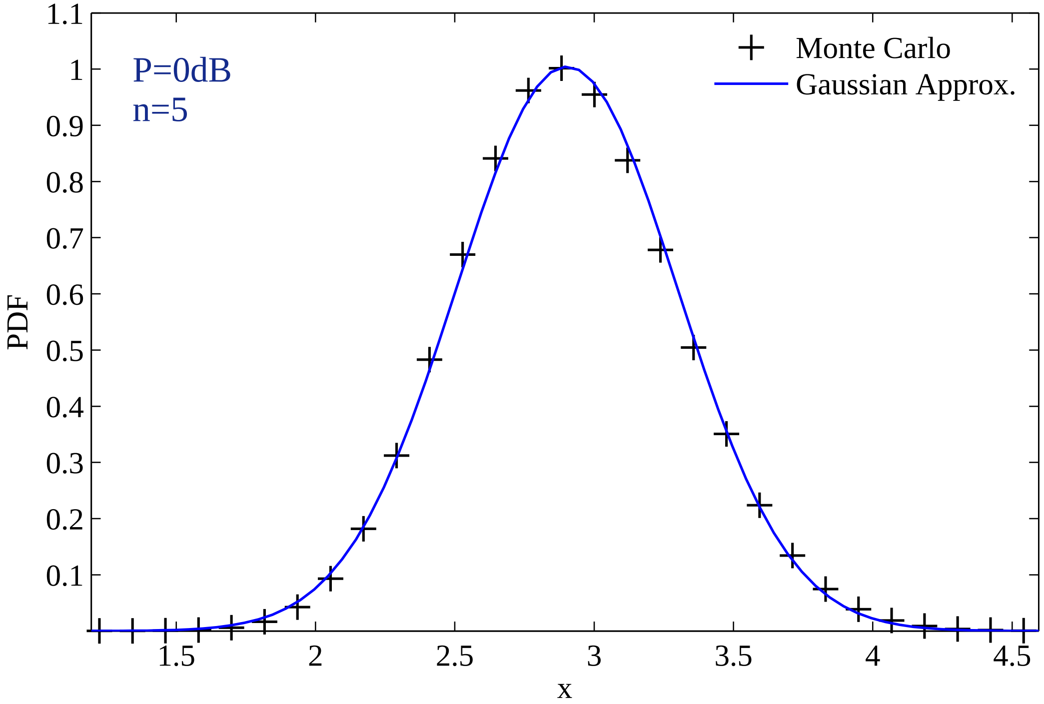
A Simple Example: MIMO Capacity Distribution (2×2)



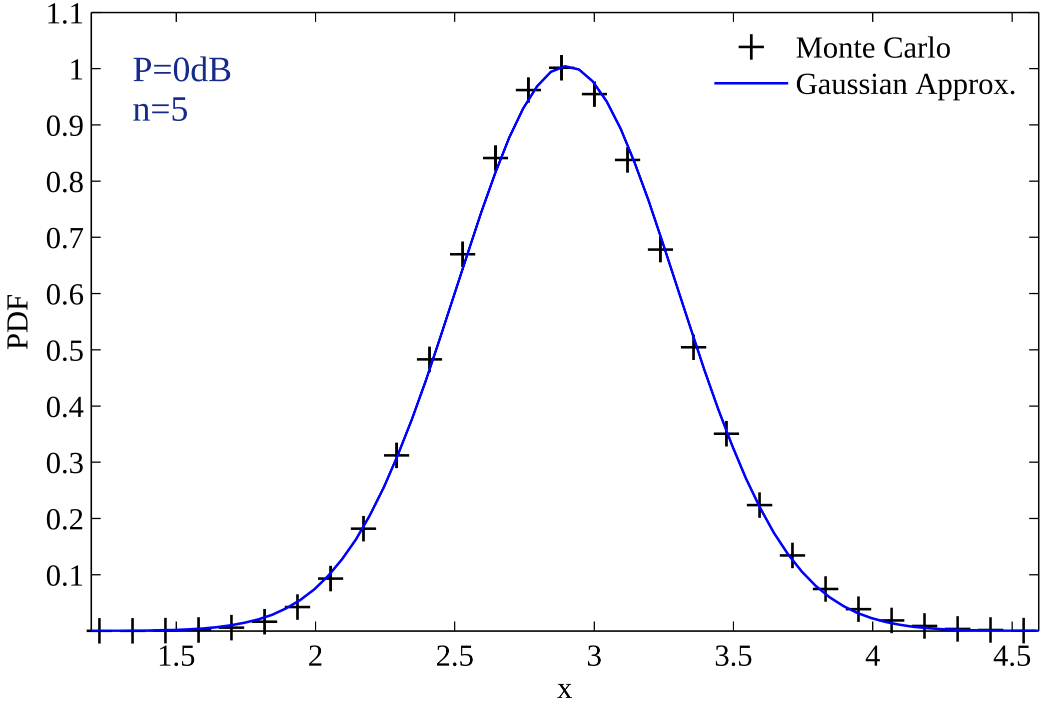
- $\ast$ Strong deviation from Gaussian as P grows!
- $\ast$ New approaches are needed to give a more "refined" asymptotic analysis (i.e., asymptotic distributions with *corrections*)

Another Example: MIMO Capacity Distribution (5×5)



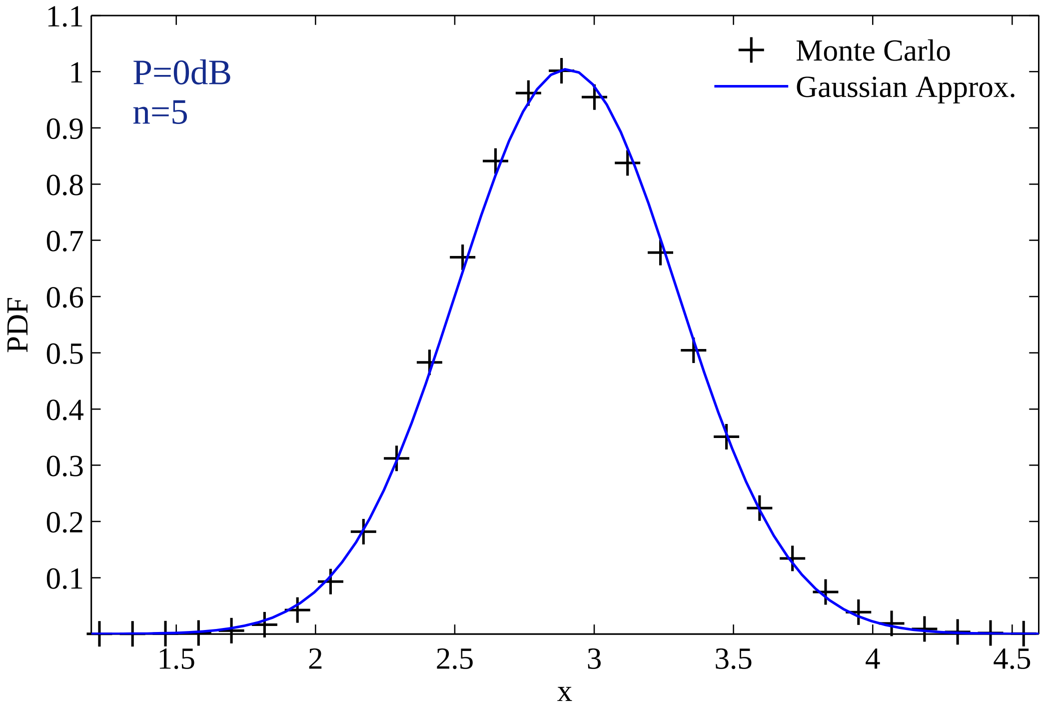
$\ast$ **A Great Fit!**

Another Example: MIMO Capacity Distribution (5×5)



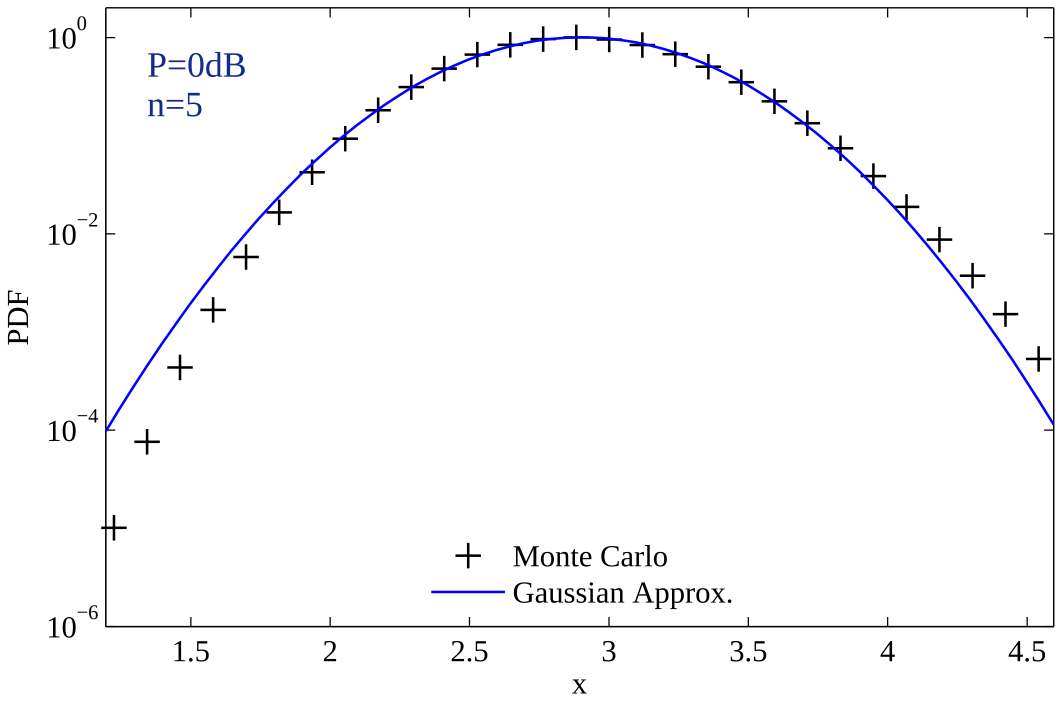
- **A Great Fit!**
- Problem solved if n sufficiently large (and P sufficiently small)?

Another Example: MIMO Capacity Distribution (5×5)



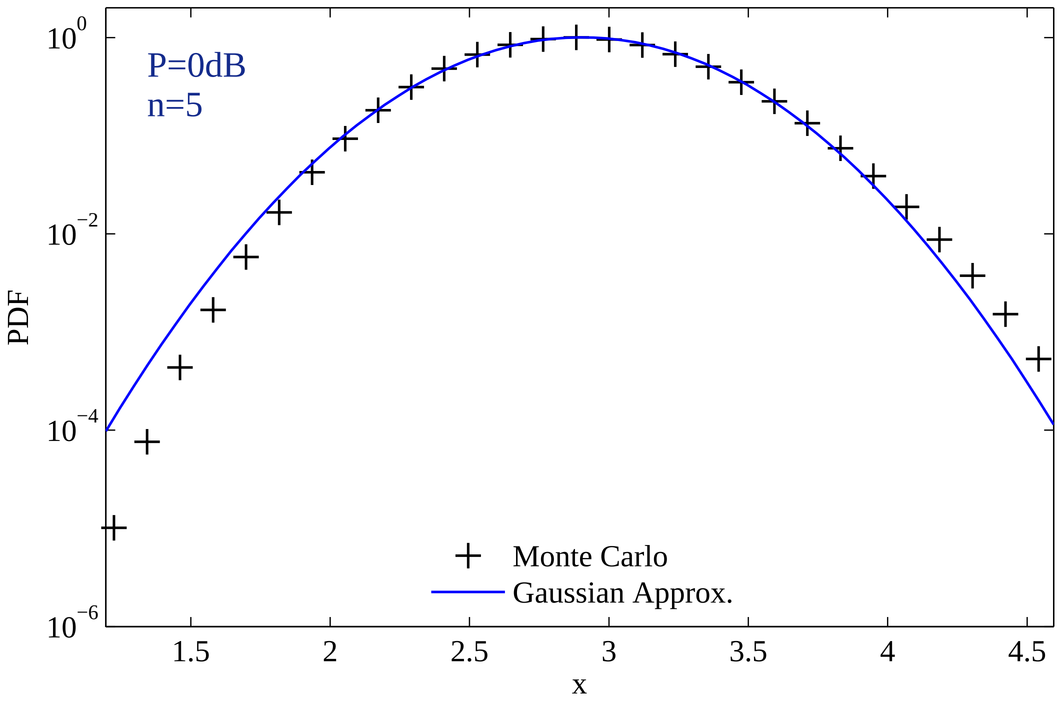
- **A Great Fit!**
- Problem solved if n sufficiently large (and P sufficiently small)?
- **Not exactly...**

Same Plot but on a LOG Scale



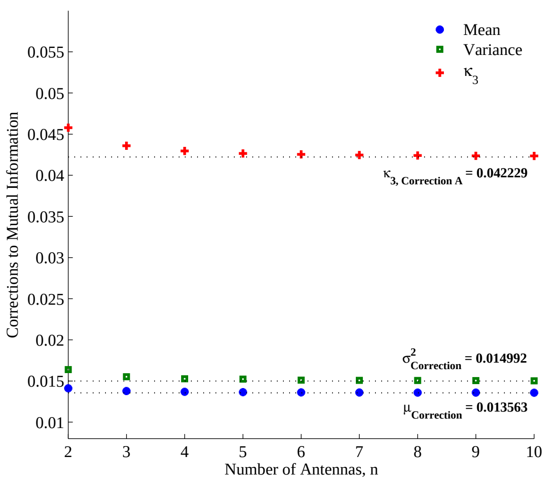
- ∗ Gaussian approx. is very **inaccurate** in the tails.
- ∗ The tails are the most important regions of interest!

Same Plot but on a LOG Scale

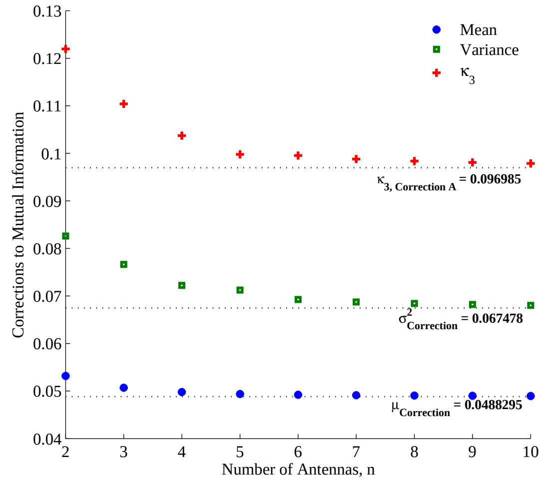


- ∗ Gaussian approx. is very **inaccurate** in the tails.
- ∗ The tails are the most important regions of interest!
- ∗ **New approaches are needed to properly capture this behavior.**

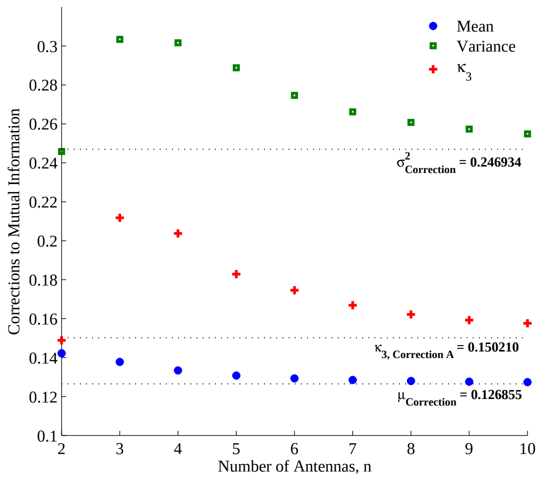
Numerical Study: Correction Terms



(a) $P = 0$ dB



(b) $P = 5$ dB

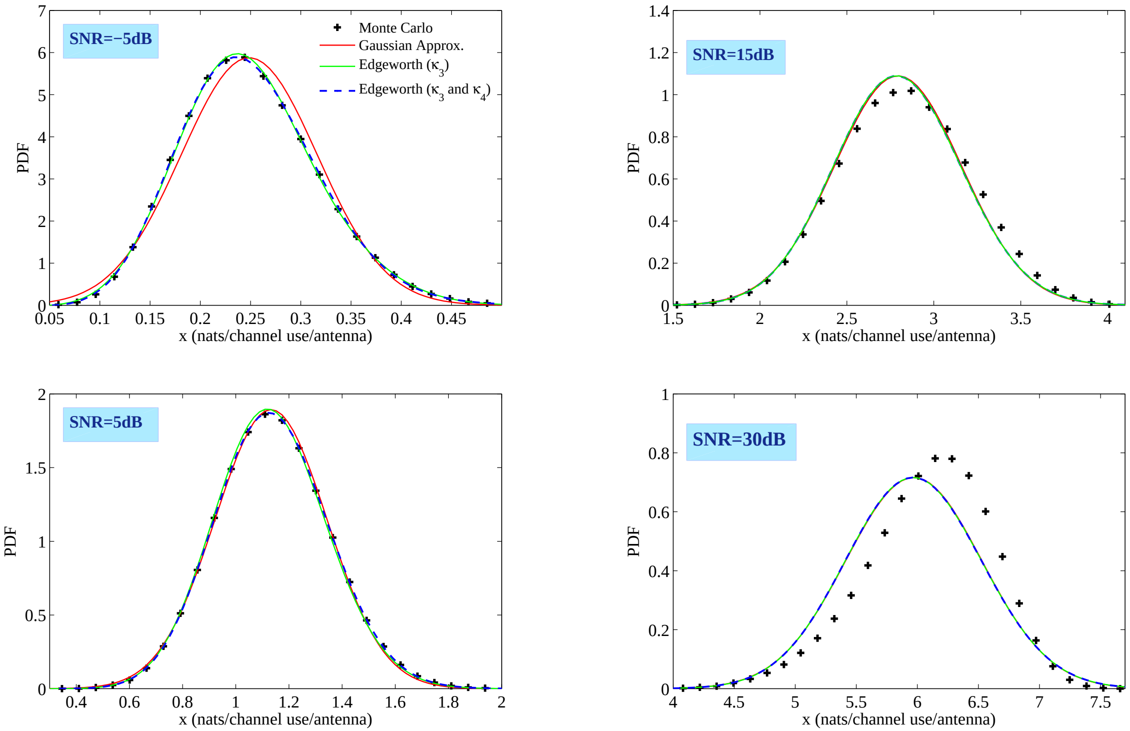


(c) $P = 10$ dB

∗ **Observations:**

- For all three cumulants, first correction term (limiting value) increases with P, as expected
- Lower order correction terms become much more significant as SNR increases
- Higher order cumulants (e.g., κ_3) are much more sensitive to SNR than the mean.

Numerical Study: 3×3 MIMO Capacity PDF



∗ **Observation:**

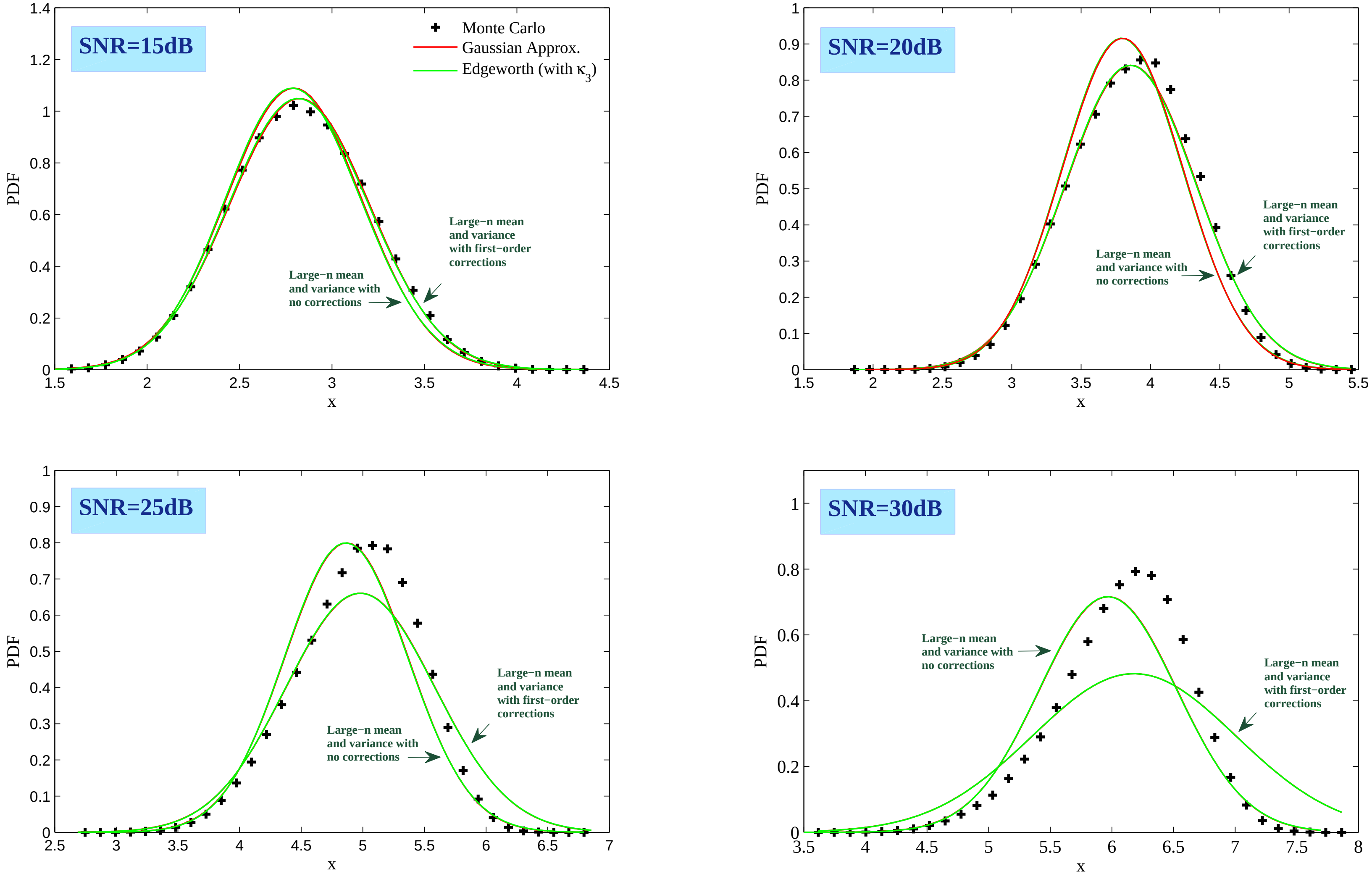
Strong deviation from Gaussian as P grows, even with higher cumulant corrections!

∗ **Key Message:**

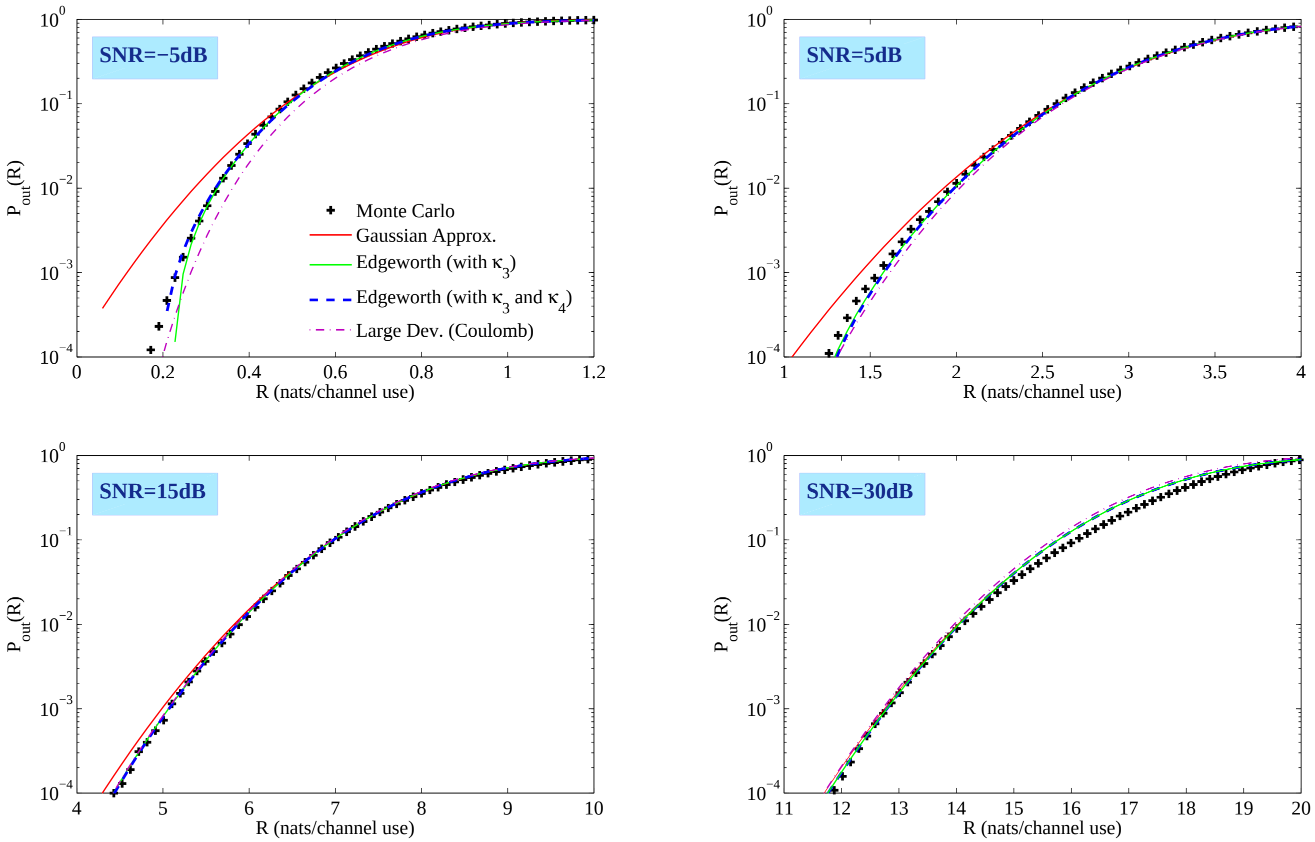
One must consider the "large n–large SNR" double-scaling to properly characterize the Gaussian deviations when P is sufficiently larger than n

This problem is still open!

Numerical Study: 3×3 MIMO Capacity PDF



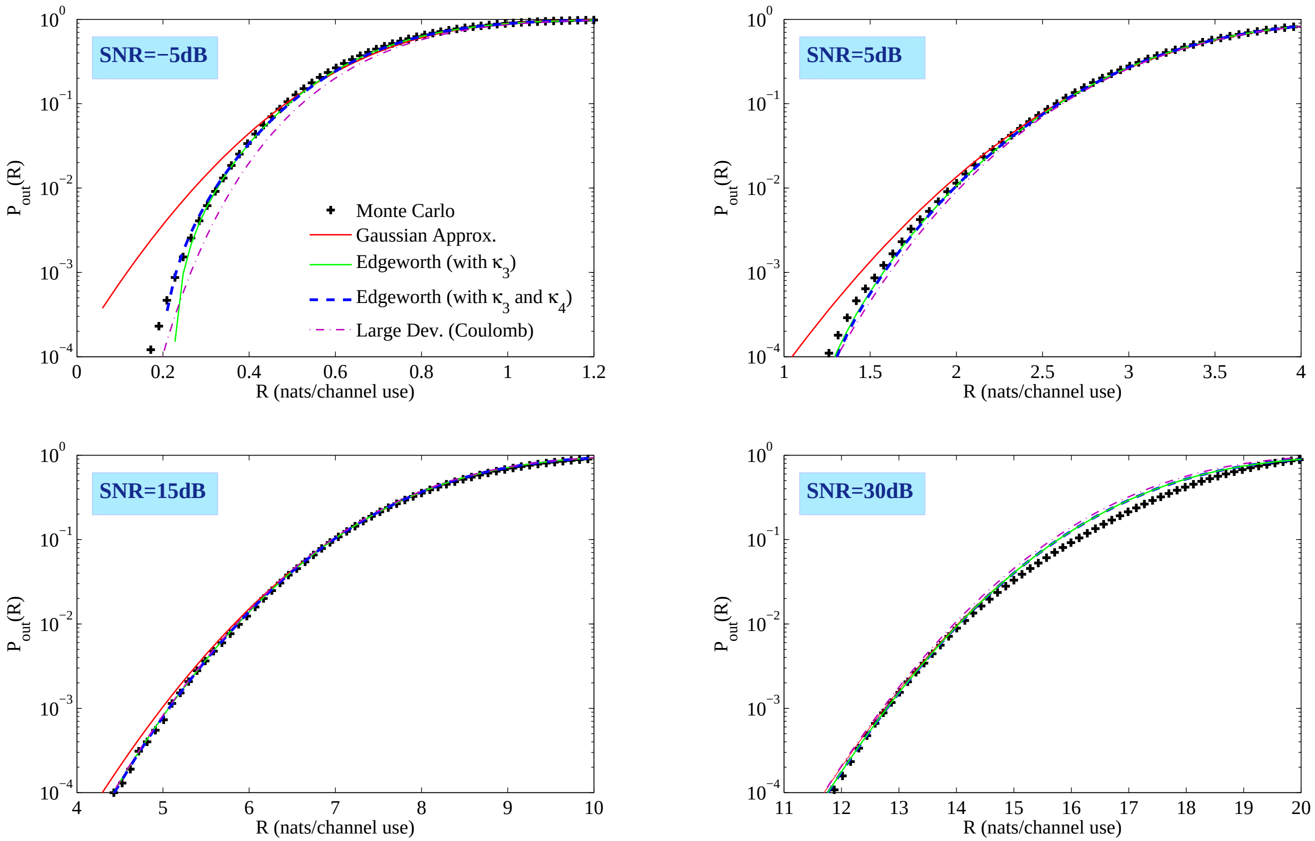
Numerical Study: 3×3 MIMO Capacity CDF



∗ **Observations:**

- Gaussian corrections significantly increase accuracy at low SNR
- As SNR increases, Gaussian is loosening around bulk, but getting more accurate around tails.

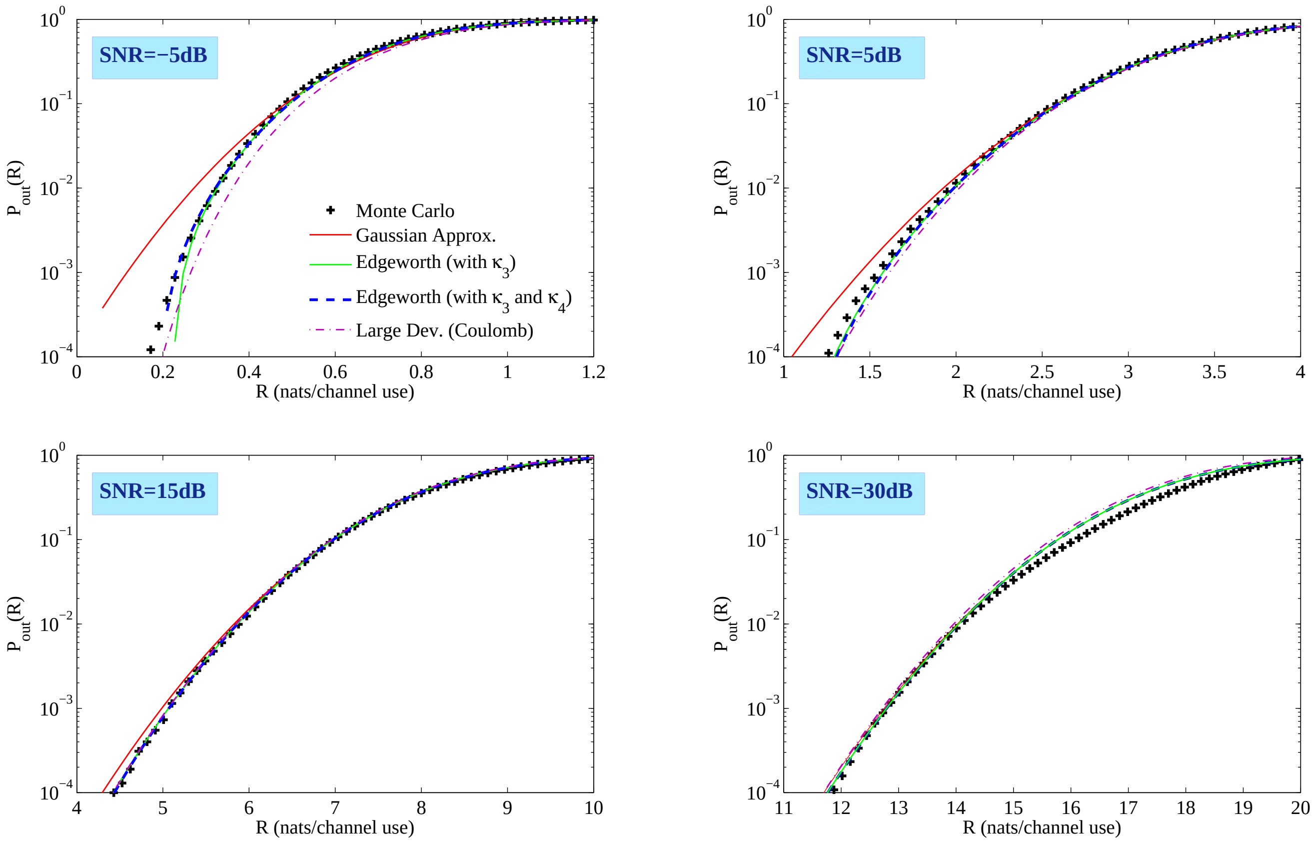
Numerical Study: 3×3 MIMO Capacity CDF



∗ **Query:**

"Why" are the tails more accurate for all curves?

Numerical Study: 3×3 MIMO Capacity CDF

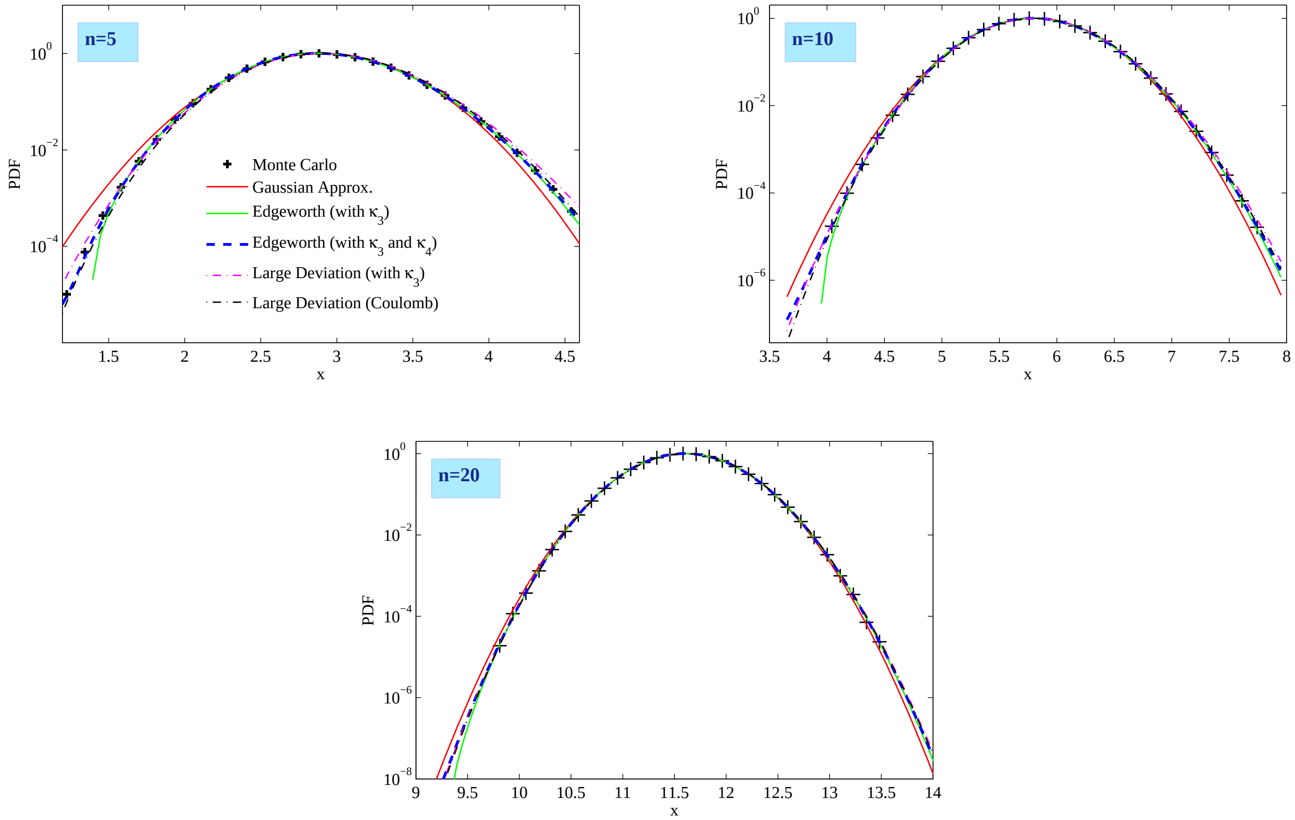


∗ **Query:**

"Why" are the tails more accurate for all curves?

Require systematic analysis of double-scaling, with SNR sufficiently large compared with n

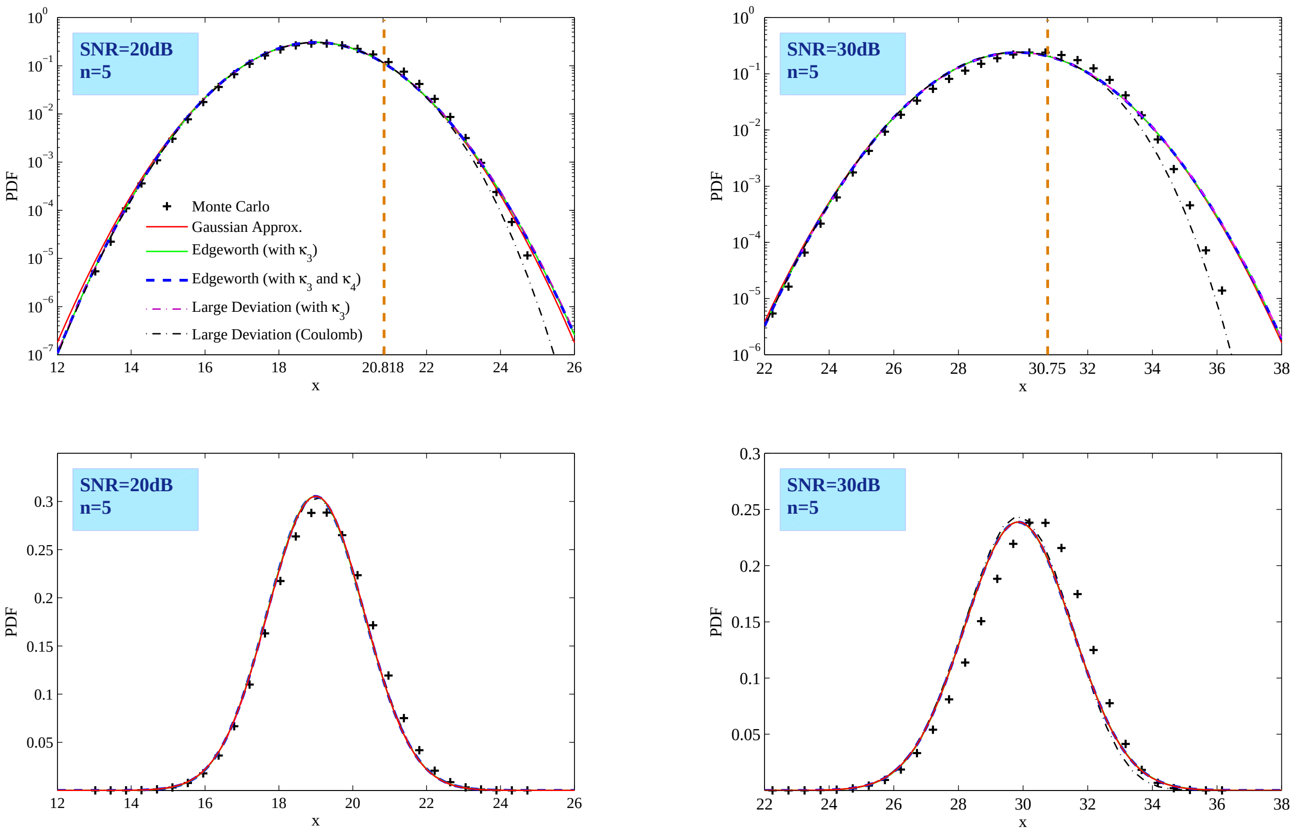
Numerical Study: MIMO Capacity CDF (SNR = 0 dB)



∗ **Observations:**

- Corrected-Gaussian (Edgeworth) very good in all cases
- Gaussian accuracy improves as n increases

Numerical Study: MIMO Capacity CDF (High SNR)



∗ **Observations:**

- Edgeworth approximations fail for high SNR in right-hand tail

- **Subtle** point: (possible) existence of **phase-transition point** causing weak singularity in distribution

- Identified in [∗] based on heuristic physics arguments

[∗] P. Kazakopoulos, P. Mertikopoulos, A. L. Moustakas, and G. Caire, “Living at the Edge: A Large Deviations Approach to the Outage MIMO Capacity”,

IEEE Trans. Inform. Theory, vol. 57, pp. 1984–2007, Apr. 2011