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Noise-induced Transition in Systems with Symmetric Absorbing States

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Absorbing States

- A state which once entered cannot be exited.
- A common feature in many non-equilibrium systems. Ubiquitous in population dynamics.
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$\longrightarrow$ Something analogous for multiple absorbing states?

Phenomenological Langevin Equation

$$\partial_t \phi = (a\phi - b\phi^3)(1 - \phi^2) + D\nabla^2\phi + \sigma\sqrt{1 - \phi^2}\,\eta(t)$$

- $\phi \in [-1, 1]$ a continuous coarse-granied variable, usually magnetisation
- proposed by Al Hammal *et al* (2005). Classifies the observed static order-disorder transitions for systems with two symmetric absorbing states.
- $\phi = \pm 1$ absorbing states.
- The shape of the potential governs the static critical behaviour.

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What effect does changing the noise strength σ have, if any?

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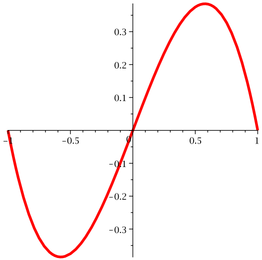
- $f(\phi_i) = \phi_i(1 - \phi_i^2)$.
- Same as force term in Langevin equation
- Pushes sites towards absorbing states at $\phi_i = \pm 1$



Microscopic Dynamics $\rightarrow$ Jump Moments $\rightarrow$ Fokker-Planck Equation $\rightarrow$ Langevin equations:

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By increasing/decreasing h, we effectively decrease/increase σ

Coarsening Behaviour

Ising-Like

- Surface tension at interface
- Curvature driven
- $\rho(t) \sim t^{-1/2}$

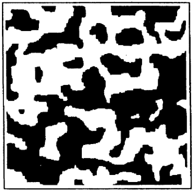


Figure: *Bray, 1994*

Voter-like

- Absence of surface tension
- Interfacial noise driven
- $\rho(t) \sim 1/\ln(t)$

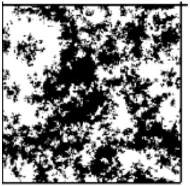


Figure: *Dornic et al, 2001*

$1/\rho(t)$ vs t

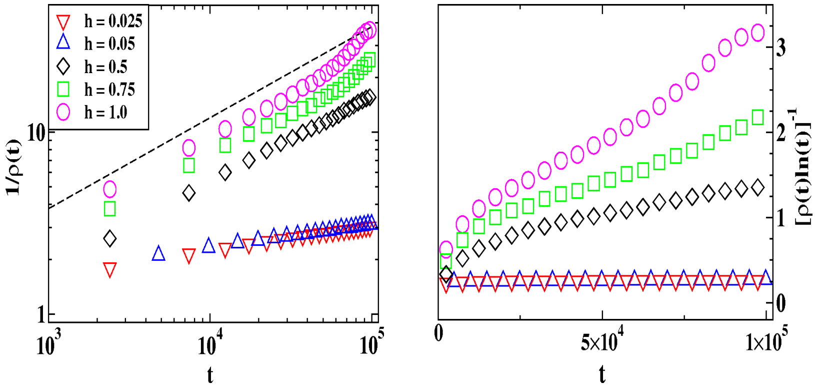


Figure: Left: log-log axes. Right: log-linear axes

Thermal Diffusion Process

Physical intuition clearer if multiplicative noise $\longrightarrow$ additive noise

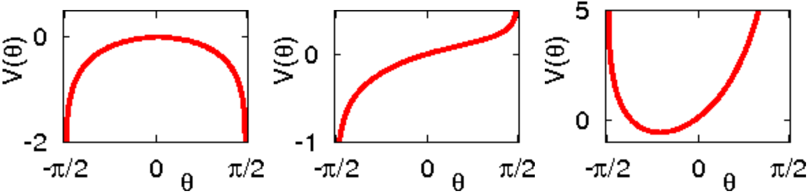
New Langevin Equations

$$\phi = \sin\theta$$

$$\dot{\theta}_i = -V'_{\mathrm{D}}(\theta_i) + \eta(t)$$

$$V_{\mathrm{D}}(\theta) = \ln\left[\left(\frac{1-\sin\theta}{1+\sin\theta}\right)^{\frac{1}{2}hzm}(\cos\theta)^{\frac{1}{2}-hz}\right] + \frac{hr}{8}\cos 2\theta$$

Diffusive Potential V_D



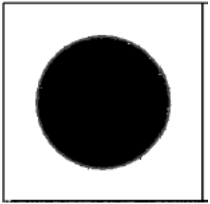
(a) $h < h_-$ (b) $h_- < h < h_+$ (c) $h > h_+$

Shape of the diffusion potential at increasing h.
$h_\pm = [2z(1 \pm \bar{m})]^{-1}$, $z = 4$ and $\bar{m} = -0.5$.

For some finite $h < h_-$ the absorbing states at $\theta = \pm\pi/2$ are readily accessible.

Lower bound on $h_- = 0.0625$ when $\bar{m} = -1$.

Droplet Experiment



Ising-like

Linear decay of droplet/magnetisation with rate $c \sim h$

Voter-like

On average, the magnetisation is conserved in the voter model.

Monte-Carlo Simulations

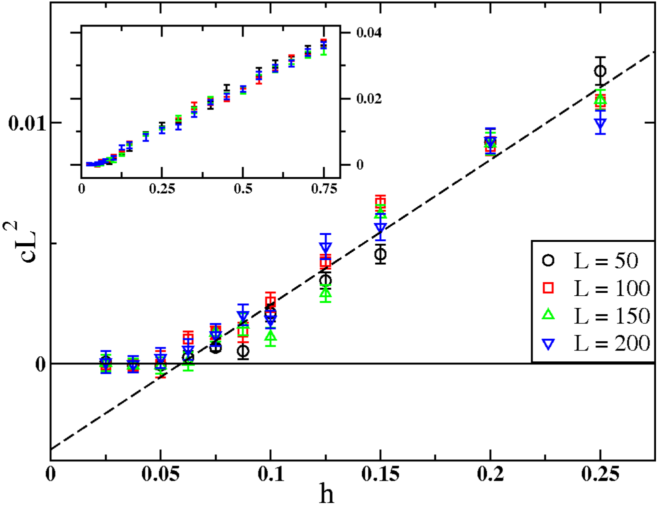


Figure: h-intercept at $h \approx 0.059$

Conclusion

- Introduced a microscopic model which explicitly allows the effect of noise strength to be studied
- At high noise strength the absorbing states become accessible; surface tension vanishes.
- Dynamic transition between coarsening regimes at some finite value of noise strength h_*
- Good agreement between theory and MC simulation

$$h_*^{th} = 0.0625, \; h_*^{sim} \approx 0.059.$$