A STOCHASTIC MODEL FOR THE DYNAMICS OF FORAGING BUMBLEBEES

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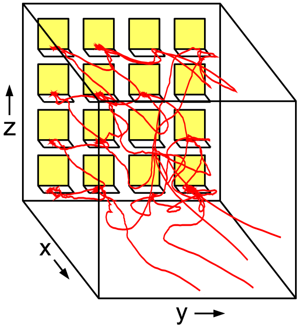
INTRODUCTION TO THE BUMBLEBEE EXPERIMENT

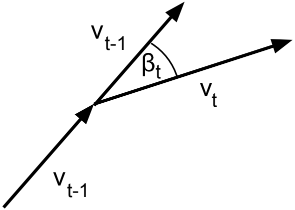


- Single Bumblebees flying in a cube (≈ 75*cm* side length)
- Trajectories of 30 bumblebee tracked by two cameras with 50fps
- Forage on artificial flowers (4x4 grid on one wall)
- Today: only interested in flights *away* from the flower-wall

EXPERIMENT: T. C. Ings and L. Chittka, Current Biology, **18(19)**, (2008)

FORAGING BEHAVIOUR UNDER PREDATION RISK: FL, Ings, Chittka, Chechkin, Klages, PRL, **108**, (2012)

REORIENTATION MODEL

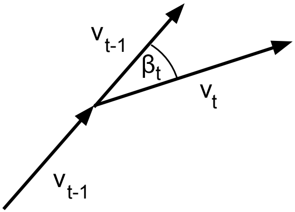


2-dimensional model by restriction to horizontal plane:
Description of the bumblebee by speed $s(t) = |v(t)|$ and turning angle β

$$\begin{aligned} \beta(t) &= \xi(t) \\ s(t) &= const \end{aligned}$$

- $\xi(t)$ drawn i.i.d. typically from a wrapped normal distribution
- Useful model as it captures directional persistence.
- Goal: a realistic model describing bumblebee foraging

GENERALIZED REORIENTATION MODEL



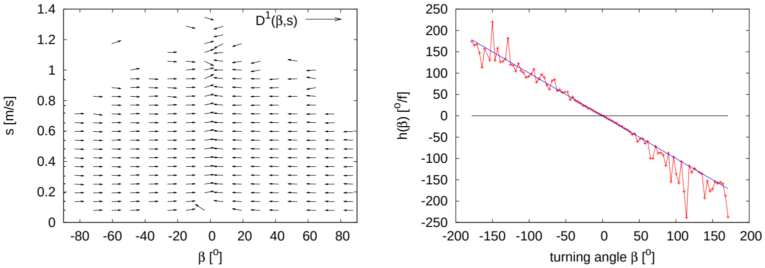
2-dimensional model by restriction to horizontal plane:
Description of the bumblebee by speed $s(t) = |v(t)|$ and turning angle β

Generalized Langevin Equation:

$$\begin{aligned} \frac{d\beta}{dt}(t) &= h(\beta(t), s(t)) + \tilde{\xi}(t) \\ \frac{ds}{dt}(t) &= g(\beta(t), s(t)) + \psi(t) \end{aligned}$$

Estimation of Drift-Coefficients

Fokker-Planck drift coefficients:

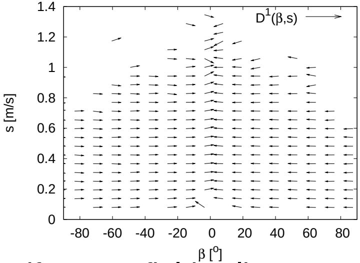


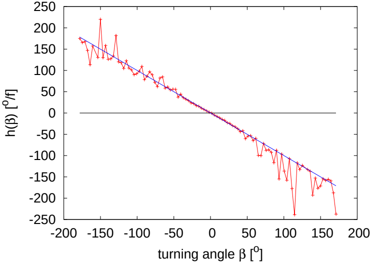
$$\frac{d\beta}{dt}(t) = h(\beta(t), s(t)) + \tilde{\xi}(t)$$

$$\frac{ds}{dt}(t) = g(\beta(t), s(t)) + \psi(t)$$

ESTIMATION OF DRIFT-COEFFICIENTS

Fokker-Planck drift coefficients:





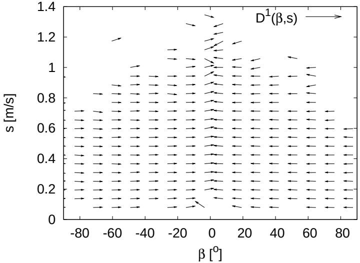
Drift vector field splits:

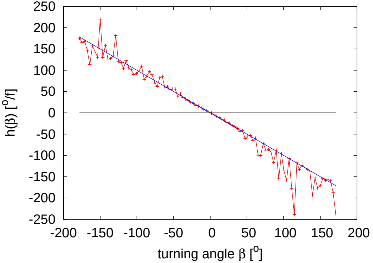
$$\frac{d\beta}{dt}(t) = h(\beta(t)) + \tilde{\xi}(t)$$

$$\frac{ds}{dt}(t) = g(s(t)) + \psi(t)$$

ESTIMATION OF DRIFT-COEFFICIENTS

Fokker-Planck drift coefficients:



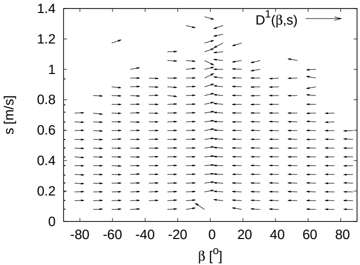


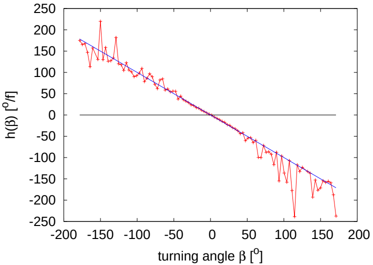
$$\frac{d\beta}{dt}(t) = -k\beta(t) + \tilde{\xi}(t)$$

$$\frac{ds}{dt}(t) = g(s(t)) + \psi(t)$$

ESTIMATION OF DRIFT-COEFFICIENTS

Fokker-Planck drift coefficients:



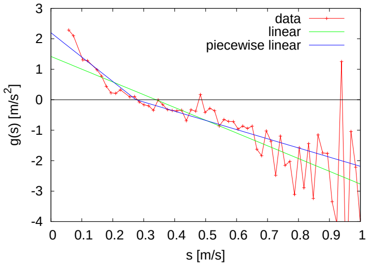


$$
\begin{aligned}
\beta(t) &= \xi(t) \\
\frac{ds}{dt}(t) &= g(s(t)) + \psi(t)
\end{aligned}
$$

VELOCITY-DRIFT

$$\beta(t) = \xi(t)$$

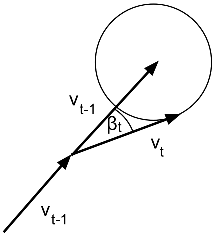
$$\frac{ds}{dt}(t) = g(s(t)) + \psi(t)$$



- Preferred speed s_0, with slower approach from above and faster from below.
- Approximation for modelling: piecewise linear drift

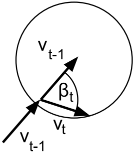
$$g(s) \approx (s - s_0) \times \begin{cases} -d_1 & \text{for} \quad s < s_0 \\ -d_2 & \text{for} \quad s \geq s_0 \end{cases} \quad \text{with } d_1 > d_2 > 0.$$

Velocity-dependent Angle-Noise



$$\beta(t) = \xi_s(t)$$
$$\frac{ds}{dt}(t) = g(s(t)) + \psi(t)$$

VELOCITY-DEPENDENT ANGLE-NOISE



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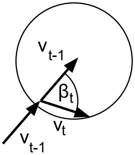
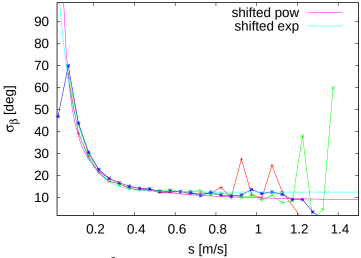




$$\beta(t) = \xi_s(t)$$

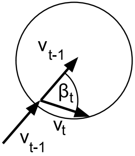
$$\frac{ds}{dt}(t) = g(s(t)) + \psi(t)$$

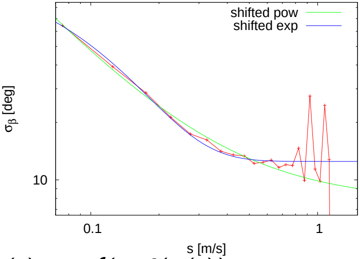
$$\xi_s(t) \sim \mathcal{N}(0, f(s(t)))$$

Candidates for f:

- $f(s) = c_1 e^{-c_2 s} + c_3$
- $f(s) = c_1 s^{-c_2} + c_3$

VELOCITY-DEPENDENT ANGLE-NOISE





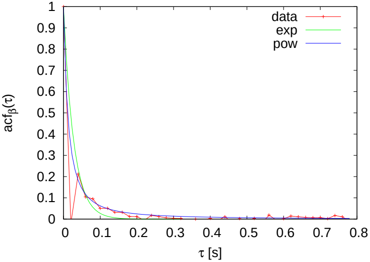
$$\beta(t) = \xi_s(t)$$
$$\frac{ds}{dt}(t) = g(s(t)) + \psi(t)$$

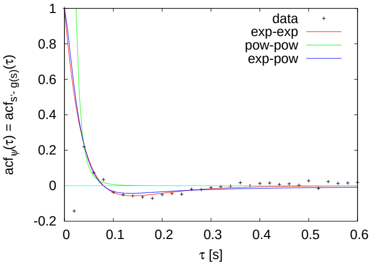
$$\xi_s(t) \sim \mathcal{N}(0, f(s(t)))$$
$$f(s) = c_1 e^{-c_2 s} + c_3$$

NOISE AUTOCORRELATIONS

Velocity-Noise shows anti-correlations:

Noise of turning angles has heavy tailed acf:





- Effect of attractive flower-wall
- Also affected by presence of predators
- Details in: FL *et al.*, PRL, **108**,098103 (2012).

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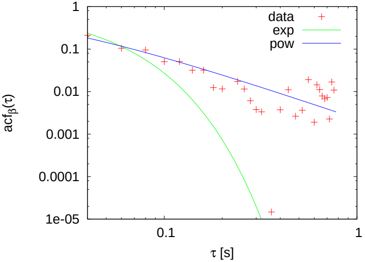
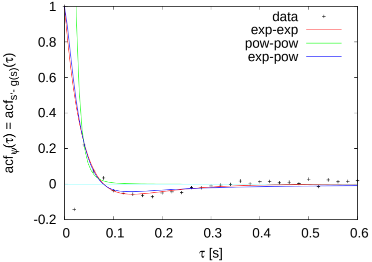




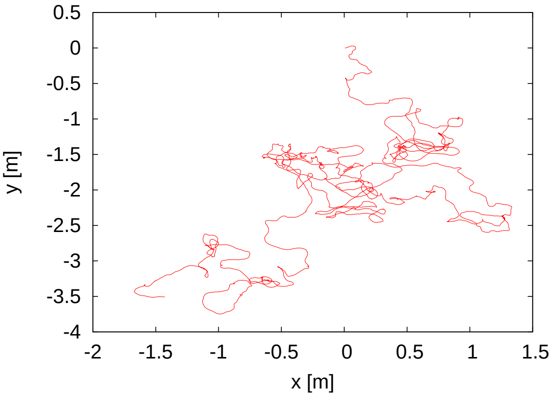
- Effect of attractive flower-wall
- Also affected by presence of predators
- Details in: FL *et al.*, PRL, **108**,098103 (2012).

COMPLETE MODEL

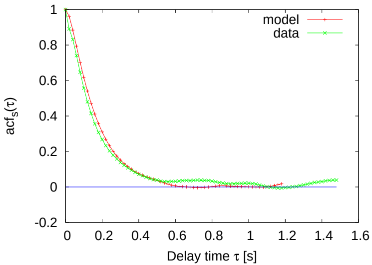
$$\begin{aligned} \beta(t) &= \xi_s(t) \\ \frac{ds}{dt}(t) &= g(s(t)) + \psi(t) \end{aligned}$$

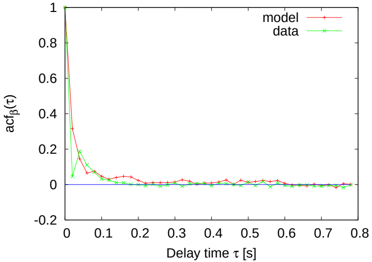
- Turning angles given by powerlaw-correlated gaussian noise $\xi_s(t) \sim \mathcal{N}(0, \sigma_\xi(s)))$ with $\sigma_\xi(s) = c_1 e^{-c_2 s} + c_3$
- Piecewise linear drift in speed:
  $g(s) \approx (s - s_0) \begin{cases} -d_1 & \text{for} \quad s < s_0 \\ -d_2 & \text{for} \quad s \geq s_0 \end{cases}$ where $d_1 > d_2 > 0$.
- ψ approx. gaussian with $\mathrm{acf}_\psi(t) \approx a e^{-\lambda_1 t} + (1 - a) e^{-\lambda_2 t}$

SIMULATION



COMPARISON OF SIMULATION AND REAL DATA





Good agreement given the number of approximations we made.

SUMMARY

- Constructed generalized reorientation model
- Found that drift vector field splits
- Discussed dependence of angle and speed
- Estimated all parameters
- Checked if resulting model is consistent with data by simulation